MEASUREMENT OF THE THERMOPHYSICAL CHARACTERISTICS BY THE METHOD OF THE REGULAR COOLING REGIME FOR A LIMITED CYLINDER WITH A SHELL

M. Marich and B. Bachlich

UDC 536.2.023

An analytic expression is used for the nonstationary temperature field of a limited circular cylinder with a shell to measure the thermal diffusivity by the method of the regular regime. The influence of a shell is taken into account by the first roots of the characteristic equations for an infinite sandwich plate and an infinite cylinder with a shell.

One of the most widespread methods for measuring the thermophysical characteristics of substances is the method of the regular regime [1]. It is used primarily for a constant medium temperature and boundary condition of the first kind. Maintenance of the medium temperature constant is of no great difficulty, and the boundary condition of the first kind ($\alpha/\lambda \to \infty$) permits avoidance of discussion on the foundation of determining the heat transfer coefficient α under nonstationary body cooling (heating) conditions in a fluid flow. Small size specimens must be used in performing the experiment.

The boundary condition of the first kind ($\alpha/\lambda \to \infty$) is realized easily for poorly conducting specimens but is quite difficult for good heat conductors. If a specimen from a good conducting material is covered by a heat-insulating substance, then the boundary condition of the first kind is also realized simply. A shell is necessary in this case, for example, even when measuring the characteristics of friable and other amorphous materials, especially when investigating damp specimens.

It should also be noted that measurement on specimens with a shell by the method of the regular regime is somewhat in the nature of universal. They can often by performed for boundary conditions of the first kind. For such measurements it is first necessary to find an analytic solution of the heat-conduction equation for a body with a shell.

Temperature Field of a Cylinder with a Shell. Specimens of cylindrical shape are often used for experimental investigations. An analytic expression for the temperature field of a cylinder with a shell and its treatment is more complex than for the field of a corresponding parallelepiped. Consequently, we consider here this problem. An analytic solution must be found for the system of equations with the initial and boundary conditions (Fig. 1)

$$\frac{\partial T_1(x, r, \tau)}{\partial \tau} = a_1\left[\frac{\partial^2 T_1(x, r, \tau)}{\partial x^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_1(x, r, \tau)}{\partial r}\right)\right], \quad 0 \leqslant x \leqslant L;\ 0 \leqslant r \leqslant R, \tag{1}$$

$$\frac{\partial T_2(x, r, \tau)}{\partial \tau} = a_2\left[\frac{\partial^2 T_2(x, r, \tau)}{\partial x^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_2(x, r, \tau)}{\partial r}\right)\right], \quad L \leqslant x \leqslant L+\delta_L;\ R \leqslant r \leqslant R+\delta_R, \tag{2}$$

$$T_1(x, r, 0) = T_2(x, r, 0) = T_0 = \text{const}, \tag{3, 4}$$

$$T_1(L, r, \tau) = T_2(L, r, \tau), \tag{5}$$

$$T_1(x, R, \tau) = T_2(x, R, \tau), \tag{6}$$

$$\partial T_1(0, r, \tau)/\partial x = 0, \tag{7}$$

$$\partial T_1(x, 0, \tau)/\partial r = 0, \tag{8}$$

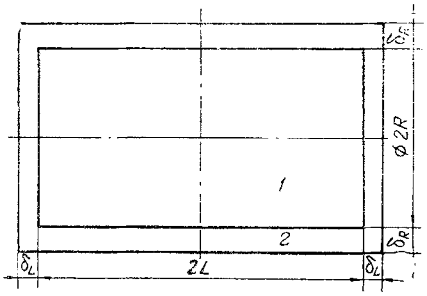
$$\lambda_1 \partial T_1(L, r, \tau)/\partial x = \lambda_2 \partial T_2(L, r, \tau)/\partial x, \tag{9}$$

$$\lambda_1 \partial T_1(x, R, \tau)/\partial r = \lambda_2 \partial T_2(x, R, \tau)/\partial r, \tag{10}$$

$$\lambda_2 \partial T_2(L+\delta_L, r, \tau)/\partial x + \alpha_L [T_2(L+\delta_L, r, \tau) - T_f] = 0, \tag{11}$$

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 43, No. 5, pp. 817-822, November, 1982. Original article submitted July 5, 1981.

 0022-0841/82/4305-1280$07.50



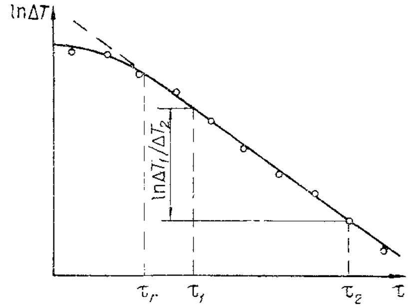


Fig. 1 Fig. 2

Fig. 1. Dimensions of a cylinder with a shell: 1) inner part of the cylinder; 2) shell.

Fig. 2. Determination of the cooling rate m = $(\ln\Delta T_1 - \ln\Delta T_2)/(\tau_1 - \tau_2)$.

$$\lambda_2 \partial T_2(x,\ R+\delta_R,\ \tau)/\partial r + \alpha_R [T_2(x,\ R+\delta_R,\ \tau) - T_f] = 0. \qquad (12)$$

In the cases $\mathrm{Bi}_L = \alpha_L L/\lambda_2 \to \infty$ and $\mathrm{Bi}_R = \alpha_R R/\lambda_2 \to \infty$ the solutions will have the following form:

$$\Theta_1(\xi,\ \eta,\ \mathrm{Fo}_R) = \frac{8}{\pi} \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} \frac{(K_L-1)^2 \sin \nu_n}{\nu_n \Psi_n/\mathrm{Bi}_L^2} \sin \nu_n \sqrt{K_a}(K_L-1) \times$$

$$\times \frac{1}{\mu_k \Phi_k/\mathrm{Bi}_R^2} \cos \nu_n \xi J_0(\mu_k \eta) \exp[-(\mu_k^2 + K_{RL}^2 \nu_n^2)\mathrm{Fo}_R],\ 0 \leqslant \xi \leqslant 1;\ 0 \leqslant \eta \leqslant 1, \qquad (13)$$

$$\Theta_2(\xi,\ \eta,\ \mathrm{Fo}_R) = \frac{8}{\pi} \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} \frac{(K_L-1)^2 \sin \nu_n \cos \nu_n}{\nu_n \Psi_n/\mathrm{Bi}_L^2} \times$$

$$\times \frac{J_0(\mu_k)}{G_0 \mu_k \Phi_k/\mathrm{Bi}_R^2} \sin \nu_n \sqrt{K_a}(K_L - \xi)[J_0(\mu_k \sqrt{K_a}\eta) Y_0(\sqrt{K_a} K_R \mu_K) -$$

$$- J_0(\sqrt{K_a} K_R \mu_K) Y_0(\mu_K \sqrt{K_a}\eta)] \exp[-(\mu_K^2 + K_{RL}^2 \nu_n^2)\mathrm{Fo}_R],$$

$$1 \leqslant \xi \leqslant K_L;\ 1 \leqslant \eta \leqslant K_R, \qquad (14)$$

where the limits are

$$\lim_{\mathrm{Bi}_L \to \infty} \nu_n \Psi_n/\mathrm{Bi}_L^2 = \nu_n (K_L-1)^2 [\sin \sqrt{K_a}(K_L-1)\nu_n \cos \sqrt{K_a}(K_L-1)\nu_n + \sqrt{K_a}(K_L-1) \sin \nu_n \cos \nu_n]; \qquad (13')$$

$$\lim_{\mathrm{Bi}_R \to \infty} \Phi_k/\mathrm{Bi}_R^2 = \mu_k \sqrt{K_a} \frac{G_1}{J_1(\mu_k)} [J_0^2(\mu_k) + J_1^2(\mu_k)] - \frac{J_0(\mu_k)}{G_0} \cdot [K_a \mu_k (G_0^2 + G_1^2) - 4/\mu_k \pi^2]. \qquad (14')$$

We obtain $\Theta_1(\xi, \eta, \mathrm{Fo}_R)$ by "intersection" (multiplication) of the solutions $\Theta_{1L}(\xi, \mathrm{Fo}_L)$ for the infinite sandwich plate and $\Theta_{1R}(\eta, \mathrm{Fo}_R)$ for the infinite cylinder with a shell [2, 3]. The expression $\Theta_2(\xi, \eta, \mathrm{Fo}_R)$ is also the "intersection" of the corresponding $\Theta_{2L}(\xi, \mathrm{Fo}_L)$ and $\Theta_{2R}(\xi, \mathrm{Fo}_R)$. Therefore, the characteristic equations for the roots ν and μ have the form

$$K_\varepsilon \,\mathrm{tg}_\nu \frac{\mathrm{tg}\sqrt{K_a}(K_L-1)\nu + \nu\sqrt{K_a}/\mathrm{Bi}_L}{1 - \dfrac{\nu\sqrt{K_a}}{\mathrm{Bi}_L}\mathrm{tg}\sqrt{K_a}(K_L-1)\nu} = 1, \qquad (15')$$

$$K_\varepsilon \,\mathrm{tg}\,\nu\, \mathrm{tg}\sqrt{K_a}(K_L-1)\nu = 1 \quad (\mathrm{Bi}_L \to \infty), \qquad (15)$$

$$G_1 J_0(\mu) = K_\varepsilon G_0 J_1(\mu) \quad (\mathrm{Bi}_R \to \infty). \qquad (16)$$

Method for Determining the Thermal Diffusivity. The temperature fields with Fourier values $\mathrm{Fo}_R > \mathrm{Fo}_R'$ are described in the regular regime by the expressions

$$\Theta_1(\xi,\ \eta,\ \mathrm{Fo}_R) \simeq A_{1L} A_{1R} K_{1L}(\xi) F_{1R}(\eta) \exp[-(\mu_1^2 + K_{RL}^2 \nu_1^2)\mathrm{Fo}_R], \qquad (17)$$

TABLE 1. Values of the Roots μ_1, ν_1

K_R	K_ε 1,70	2,00	2,40
	$K_a = 4,0$		
1,004	2,3724	2,3667	2,3592
1,008	2,3406	2,3295	2,3148
1,012	2,3094	2,2931	2,2716
1,018	2,2639	2,2403	2,2094
	$K_a = 9,0$		
1,004	2,3564	2,3480	2,3368
1,008	2,3092	2,2922	2,2714
1,012	2,2635	2,2398	2,2088
1,018	2,1975	2,1639	2,1207

$$\Theta_2(\xi,\ \eta,\ \mathrm{Fo}_R) \simeq B_{1L}B_{1R}f_{1L}(\xi)f_{1R}(\eta)\exp[-(\mu_1^2 + K_{RL}^2\nu_1^2)\mathrm{Fo}_R]. \quad (18)$$

Then we can write

$$\frac{\partial \ln \Theta_1(\xi,\ \eta,\ \mathrm{Fo}_R)}{\partial \mathrm{Fo}_R} = (\mu_1^2 + K_{RL}^2\nu_1^2),$$

$$\frac{\partial \ln |T_1(x,\ r,\ \tau) - T_f|}{\partial \tau} = -a_1(\mu_1^2/R^2 + \nu_1^2/L^2) = -m. \quad (19)$$

The temperature difference $\Delta T = T_1 - T_f$ is measured at the times τ_1 and τ_2 in test [1] and the cooling time m is thereby found by using Fig. 2, and by using the body shape factor K ($= K_\infty$) an expression can be written to determine the thermal diffusivity

$$a_1 = mK = \frac{m}{\mu_1^2/R^2 + \nu_1^2/L^2} \quad (20)$$

The shell influences the magnitude of the cooling rate m directly by means of its thermophysical and geometric characteristics. It implicitly affects the shape factor K in terms of the first roots of the characteristic equations (15) and (16) since they are functions of the following kind:

$$\nu_1 = \nu_1(K_a,\ K_\varepsilon,\ K_L), \quad (21)$$

$$\mu_1 = \mu_1(K_a,\ K_\varepsilon,\ K_R). \quad (22)$$

The roots ν_n of (15) and even of the more complex (15') can be evaluated by using a small electronic computer. Unfortunately, this is impossible with respect to the calculation of the roots μ_1 of (16). Hence, a table must be compiled of the roots μ_1 for sufficiently large ranges of variation of the parameters in (22). Part of the table the authors obtained is presented as an example.

Determination of the Thermal Diffusivity a_1. The effective method of calculation is exhibited in the following example: a clay specimen, moistness dry u = 6.2944%, density ρ_1 = 2.123 kg/m^3 (measured by independent test), specific heat c_1 = 1.006 kJ/kg·K (from the literature), 2L = 94.71143 mm, 2R = 47.125 mm,* paraffin shell λ_2 = 0.2442 W/m·K, ρ_2 = 910 kg/m^3, c_2 = 2.47* kJ/kg·K, δ_L = 0.4026 and 0.64 mm, δ_R = 0.19625 and 0.32 mm, m = 18.775 and 16.952 h^{-1}.

Performing the calculations, we find $K_L = 1 + \delta_L/L = 1.0085$, $K_R = 1.00416$, $K_a = 910 \cdot 2470\, a_1/0.2442$, $K_\varepsilon = 1.9727\sqrt{\lambda_1}$. Assuming $a_1 = 4.95 \cdot 10^{-7}$ m^2/sec (a rough estimate for clay), we obtain $K_a = 4.5561$, $K_\varepsilon = 2.0284$.

By linear interpolation for K_R = 1.004 and 1.008 for K_ε = 2.00 and 2.40 and then by the same interpolation between two K_ε we obtain μ_1' = 2.3590 (Table 1, K_a = 4.0). Performing the same interpolation in the table K_a = 9.0, we obtain μ_1 = 2.3397. The numbers μ_1', μ_1'' must be interpolated in the ratio $\sqrt{4}$, $\sqrt{4.5561}$ and $\sqrt{9}$, and then μ_1 = 2.3564. We find ν_1 = 1.51508 from (15), and then from (20)

*The geometric data are the statistical means, the paraffin characteristics are taken from the literature at ∿25°C.

$$K = \frac{0.25}{(2.3564/0.047125)^2 + (1.51508/0.09471143)^2} = 9.0704 \cdot 10^{-5}\ \mathrm{m}^2;$$

$$a_1 = mK = 5.21528 \cdot 10^{-3} \cdot 9.0704 \cdot 10^{-5} = 4.73047 \cdot 10^{-7}\ \mathrm{m^2/sec}.$$

This value is 4.5% less than that assumed. Hence, the calculation should be repeated. Repeating the calculation for $a_1 = 4.73047 \cdot 10^{-7}$, we obtain the second value $a_1 = 4.7045 \cdot 10^{-7}$. Finally, the third time for $a_1 = 4.7045 \cdot 10^{-7}$ we have $a_1'' = 4.714 \cdot 10^{-7}$ and finally $a_1 = (a_1' + a_1'')/2 = 4.709 \cdot 10^{-7}$ m²/sec, $\lambda_1 = a_1 \rho_1 c_1 = 1.006$ W/m·K.

In a test with a thicker shell ($K_L = 1.01351$, $K_R = 1.0131358$), a cooling rate of $m = 4.9375 \cdot 10^{-3}$ sec^{-1} was observed. Repetition of the calculation procedure yields $a_1 = 4.7983 \cdot 10^{-7}$ m²/sec ($\lambda_1 = 1.0249$ W/m·K), whose magnitude is 1.9% greater than the transition value. This indicates the good agreement between the experimental and theoretical results.

NOTATION

T, temperature; τ, time; x, r, cylindrical coordinates; $\Theta = (T(x, r, \tau) - T_f)/(T_0 - T_f)$; L, R, δ, cylinder dimensions (Fig. 1); $\xi = x/L$; $\eta = r/R$; $\mathrm{Fo}_R = a_1\tau/R^2$; $\mathrm{Fo}_L = a_1\tau/L^2$; a, thermal diffusivity; λ, heat conduction; ρ, density; c, specific heat; α, heat-transfer coefficient; $K_a = a_1/a_2$; $K_\varepsilon = \sqrt{\lambda_1\rho_1 c_1/\lambda_2\rho_2 c_2}$; $\mathrm{Bi}_L = \alpha_L L/\lambda_2$; $\mathrm{Bi}_R = \alpha_R R/\lambda_2$; $K_L = 1 + \delta_L/L$; $K_R = 1 + \delta_R/R$; $K_{RL} = R/L$; J_0, J_1, Y_0, Y_1, Bessel functions; ν_n, μ_k, roots of the characteristic equations (15) and (16); $G_0 = J_0(\mu\sqrt{K_a})Y_0(\mu K_R\sqrt{K_a}) - J_0(\mu K_R\sqrt{K_a})Y_0(\mu\sqrt{K_a})$ $G_1 = J_1(\mu\sqrt{K_a})Y_0(\mu K_R\sqrt{K_a}) - J_0(\mu K_R\sqrt{K_a})Y_1(\mu\sqrt{K_a})$. Subscripts: 1, main cylinder, first; 2, shell; L, axial, plate; R, radial, cylinder; 0, initial; f, fluid medium.

LITERATURE CITED

1. G. M. Kondrat'ev, Thermal Measurements [in Russian], Mashgiz, Moscow—Leningrad (1957).
2. H. Tautz, Wärmeleitung-Temperaturausgeich, Akademie-Verlag, Berlin (1971).
3. A. V. Lykov, Theory of Heat Conduction [in Russian], Vysshaya Shkola, Moscow (1967).

THEORETICAL AND EXPERIMENTAL INVESTIGATION OF THE ABSORPTION COEFFICIENT OF DIFFERENT BLACKBODY MODELS

S. B. Kiselev and O. A. Kraev

UDC 536.3

Numerical computations are performed and results are presented of an experimental investigation of the effective absorption coefficient of radiant heat flows of cavities of complex configuration.

The measurement of radiant energy by using thermal detectors—radiometers usually includes two stages: absorption of radiant energy and its conversion into heat, and measurement of the quantity of absorbed heat. Blackbody models in the form of spherical, cylindrical, conical cavities, as well as more complex configurations are used as radiation absorbers in precision radiometers. The computations of the cavity absorption coefficients are fraught with serious technical difficulties and are executed principally for simple shapes [1-4]. Papers [5, 6] are also known in which an attempt is made to analyze absorbers of more complex configuration under definite simplifying assumptions. The lack of experimental work in this area does not permit an assessment of the legitimacy of the assumptions made and of the accuracy of the results obtained. In this paper we present the results of a complex investigation in which the absorption coefficients of cavities of certain complex shapes used in practice were determined by both computational and experimental means.

The problem of theoretical computational determination of the absorption coefficient of combination cavities consisting of N different surfaces by using a generalized zonal method will reduce, in the long run, to solving a system of integral equations of the form [7]

All-Union Scientific-Research Institute of Physicotechnical and Radiotechnical Measurements.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 43, No. 5, pp. 822-826, November, 1982. Original article submitted July 14, 1981.